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# CALCULUS.

## NOTE ON PROBLEM 84, BY DR. E. WOELFFING, STUTTGART, GERMANY.

The solution of question 84 can be found in some theorems proved by W. Merkelbach (*Ueber Rollkurven welche von einer Graden eingehüllt werden*, Diss. Marburg, 1881). The first of them is the following :

If a curve,  $C$ , rolls upon another curve,  $C'$ , and a point,  $P$ , in the plane of  $C$  describes a straight line  $L$ , and we make afterwards the curve  $C'$  roll upon the curve  $C$ , then a straight line,  $L$ , in the plane of  $C'$  will always pass through  $P$  (page 18 of the paper quoted).

Now (and this is the second theorem of Merkelbach) if a sinusoid rolls upon an ellipse, a straight line in the plane of the former passes through a focus of the latter (page 24) ; therefore, if the ellipse rolls upon the sinusoid, any one of the foci of the former will describe a straight line.

Stuttgart, Germany, July 19, 1899.

91. Proposed by GUY B. COLLIER, Schenectady, N. Y.

Find the area of a loop of the curve  $r^2 \cos \theta = a^2 \sin 3\theta$ . [From Hall's *Differential and Integral Calculus*].

I. Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; M. C. STEVENS, A. M., Professor of Mathematics, Purdue University, Lafayette, Ind.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; ELMER SCHUYLER, Reading, Penna.; and J. SCHEFFER, A. M., Hagerstown, Md.

The curve has two equal loops, one in the first and the other in the third quadrant.

The limits of  $\theta$  are 0 and  $\frac{1}{2}\pi$ .

$$\begin{aligned} \therefore A &= \frac{1}{2} \int_0^{\frac{1}{2}\pi} r^2 d\theta = \frac{1}{2} a^2 \int_0^{\frac{1}{2}\pi} \frac{\sin 3\theta d\theta}{\cos \theta} \\ &= \frac{1}{2} a^2 \int_0^{\frac{1}{2}\pi} (4 \sin \theta \cos \theta - \tan \theta) d\theta = \frac{1}{2} a^2 (3 - 2 \log 2). \end{aligned}$$

II. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; and GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.

The shape of the curve is seen in the diagram.

The limits are evidently from  $0^\circ$  to  $30^\circ$  for a loop.

$$\begin{aligned} \text{Then } A &= \frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int_0^{\frac{1}{2}\pi} \frac{a^2 \sin 3\theta}{\cos \theta} d\theta \\ &= \frac{a^2}{2} \int_0^{\frac{1}{2}\pi} \frac{3 \sin \theta - 4 \sin^3 \theta}{\cos \theta} d\theta = \frac{a^2}{2} \int_0^{\frac{1}{2}\pi} 3 \tan \theta d\theta \\ &\quad - 2a^2 \int_0^{\frac{1}{2}\pi} (\tan \theta - \sin \theta \cos \theta) d\theta \end{aligned}$$

